ICARUS 107, 388–403 (1994)

Lunar Crescent Visibility

LEROY E. DOGGETT

Nautical Almanac Office, U.S. Naval Observatory, Washington DC 20392
E-mail: doggett@ariel.usno.navy.mil

AND

BRADLEY E. SCHAEFER[1]

Goddard Space Flight Center, Code 661, Greenbelt, Maryland 20771

Received August 13, 1993; revised November 22, 1993

We report the results of five Moonwatches, in which more than 2000 observers throughout North America attempted to sight the thin lunar crescent. For each Moonwatch we were able to determine the position of the Lunar Date Line (LDL), the line along which a normal observer has a 50% probability of spotting the Moon. The observational LDLs were then compared with predicted LDLs derived from crescent visibility prediction algorithms. We find that ancient and medieval rules are highly unreliable. More recent empirical criteria, based on the relative altitude and azimuth of the Moon at the time of sunset, have a reasonable accuracy, with the best specific formulation being due to Yallop. The modern theoretical model by Schaefer (based on the physiology of the human eye and the local observing conditions) is found to have the least systematic error, the least average error, and the least maximum error of all models tested. Analysis of the observations also provided information about atmospheric, optical and human factors that affect the observations. We show that observational lunar calendars have a natural bias to begin early. © 1994 Academic Press, Inc.

1. INTRODUCTION

The visibility of the thin lunar crescent just after New Moon has considerable historical and cultural importance, since many societies have used the lunar phase cycle as the basis for their calendars. As a result, a correct interpretation of historic records sometimes necessitates an understanding of lunar crescent visibility. Even today a variety of cultures still begin their months with the first sighting of the crescent Moon, including the 22% of the world population that holds the Islamic faith. In addition to observational calendars, the question of lunar visibility also is embedded in such formalized calendric systems as the Hebrew calendar and the Gregorian rules for calculating Easter. Complementing this cultural importance is the enjoyable challenge of sighting the beautiful young crescent. For astronomers this has led to a friendly competition to sight the youngest possible Moon.

Serious attempts to create mathematical rules for predicting first visibility date back at least to the Babylonian civilization (Ilyas 1984). Before 1977, however, all efforts were empirical in nature, with a criterion being chosen to satisfy a limited and local set of observations. During the Middle Ages, Islamic astronomers worked on the prediction problem, resulting in the creation of many tables predicting first visibility as a function of the Moon's position with respect to the Sun. Since then, the only significant empirical work has been by Fotheringham (1910), who deduced a criterion based on the altitude and the azimuth of the Moon at the time of sunset. These empirical rules are all plagued by the implicit assumption that all observing sites have the exact same observing conditions. We can scarcely expect a single prediction criterion to work successfully for the swamps of Louisiana and the mountain tops of Arizona or for summer and winter conditions over most of the world.

In an effort to overcome this fundamental difficulty, Bruin (1977) pioneered a new method based on accurate modeling of all relevant factors. He considered the physiology of the human eye, the brightness of the twilight sky, the extinction in the atmosphere, and the surface brightness of the Moon. Unfortunately, he made a number of grossly incorrect assumptions: the assumed lunar surface brightness is many orders of magnitude in error; sky brightness during twilight is considered to depend only on the angle of the Sun below the horizon; physiological data are not corrected for color, pupil diameter, or binocular vision; visibility of the unevenly illuminated crescent is equated with the visibility of a uniform disk a hundred times smaller than the illuminated portion of the Moon; and no attempt is made to account for the local observing

[1] Research Scientist, Universities Space Research Association.

0019-1035/94 $6.00

conditions. Nevertheless, Bruin must be credited with great insight in deriving a physical model that would account for all variations in observing conditions.

Recently, one of us (B.E.S.) has worked on Bruin's theoretical method to correct its many deficiencies. The result is a computer program (Schaefer 1990b) that calculates the probability of crescent visibility for any given date and location. This was tested against 201 observations collected from the scientific literature (Schaefer 1988b).

Although the Schaefer collection of observations provided an important database for testing models, the observations are scattered in both time and location. Thus many important questions could not be answered. In 1987, one of us (L.E.D.) realized that a massive observing campaign by observers widely spread over a large area could efficiently and decisively answer many vital questions regarding crescent visibility (Doggett and Seidelmann 1988). For logistical reasons, these Moonwatch campaigns were organized in North America. We have now run five such campaigns. These have increased the observational database by an order of magnitude. We report our use of this database in testing prediction models and studying factors that affect visibility.

2. THE LUNAR DATE LINE (LDL)

The historic prediction models were concerned with predicting the time and location at which first sighting would occur. Taking a global approach, Ilyas (1978, 1981, 1984) proposed a new concept for systematizing a lunar calendar, the LDL. This concept is similar to the solar International Dateline. However, whereas the solar dateline divides the world into different solar days, the LDL divides the world into different lunar months. To the west of the LDL the crescent is visible and the lunar month starts that night, while to the east of the LDL the crescent is not visible and the lunar month does not start until the next evening.

Ilyas recognized the existence of a "zone of uncertainty," a region centered along the LDL in which the actual visibility of the Moon is uncertain. This zone results from unpredictable and unknowable variations in local observing conditions and abilities of observers. Within the zone of uncertainty, the visibility of the Moon cannot be predicted with certainty, whereas outside the zone of uncertainty the prediction is made with high confidence. Thus, the visibility of the crescent within the zone of uncertainty is probabilistic in nature. We take the LDL to be the locus of positions for which the probability of seeing the crescent is 50%. The boundaries of the zone of uncertainty depend on the chosen confidence level for sighting the Moon.

As presented above, the LDL and zone of uncertainty are determined by observations. However, they can also be associated with prediction algorithms. For example, if a prediction criterion specifies that the Moon will be visible if it sets 48 min after the Sun, a curve in longitude and latitude can be calculated to define a predicted LDL. By comparing the observed and predicted LDLs, we can test the accuracy of prediction algorithms.

3. MOONWATCH OBSERVATIONS

The dates of our five Moonwatches were 1987 April 28, 1988 July 14, 1989 April 6, 1989 May 5, and 1990 August 21, hereafter designated Moonwatches 1, 2, 3, 4 and 5, respectively. Observers were recruited by three methods. First, for all five Moonwatches, we asked 20–50 friends in the astronomical community to make observations. This was the primary mode of recruitment for Moonwatches 1 and 4. Second, we recruited observers for Moonwatches 2, 3, and 5 by direct appeals in *Sky & Telescope* magazine (Doggett et al. 1988; Doggett and Schaefer 1989, 1990). Third, the U.S. Naval Observatory distributed news releases announcing Moonwatches 2 and 5. These were widely disseminated throughout the United States by the news media.

The five Moonwatches had responses involving 78, 1463, 662, 35, and 230 observers, respectively. Some of the reports represent multiple observers at the same location. Such a group report cannot be treated as a set of independent observations, but must be considered to be a single observation. In addition, we received many reports by observers who were rained out, clouded out, or who looked at the wrong time. When all such reports are consolidated or eliminated, we have a total of 23, 982, 387, 5, and 93 independent reports, respectively, for the five Moonwatches. We also have 44 additional observations (involving 54 observers) not related to an organized Moonwatch. In all, this paper is reporting on 1534 independent, useful observations involving 2522 observers.

The minimum required information for each report was the observer's name and location, and whether the Moon was seen. However, we encouraged additional information and published a special form in *Sky & Telescope* for its readers (Doggett *et al.* 1988). Almost all reports contained some information about observing conditions. The majority of the reports also indicated the age of the observer, the relative humidity and temperature, the exact times of visibility, and comments on various special conditions. Roughly a quarter of the reports provided enough information that we could estimate the experience and acuity of the observer. About a third of the observers reported looking for the Moon with binoculars or telescopes.

The individual observations for Moonwatch 1 were published by Doggett and Seidelmann (1988) and Schaefer

TABLE I
Moonwatch 2 (1988 July 14): Percentage of Observers Reporting Sightings

	Percentage	No. of observers
Atlantic		
Connecticut	100	1
Delaware	0	1
Florida	33	3
Georgia	0	6
Massachusetts	33	3
Maryland	0	19
North Carolina	0	11
New Jersey	0	6
New York	13	8
Pennsylvania	12	34
Quebec	33	3
South Carolina	0	1
Virginia	6	16
Washington, DC	0	5
	9	117
Northern Plains		
Colorado	39	49
Kansas	40	5
Manitoba	0	1
Montana	17	6
Nebraska	0	4
Saskatchewan	0	1
South Dakota	0	7
Wyoming	22	9
	29	82
Pacific Northwest		
Alberta	25	8
British Columbia	0	1
Idaho	22	27
Oregon	57	7
Washington	19	26
	25	69
Midwest		
Iowa	20	15
Illinois	28	40
Indiana	33	9
Michigan	21	58
Minnesota	0	7
Missouri	10	10
Ohio	21	14
Ontario	20	25
Wisconsin	27	62
	23	240
South		
Alabama	33	3
Arkansas	22	9
Kentucky	0	1
Louisiana	57	14
Mississippi	22	9
Tennessee	0	4
	33	40
Southern Plains		
New Mexico	100	5
Oklahoma	54	26
Texas	78	95
	74	126
Southwest		
Arizona	86	101
California	80	169
Nevada	64	14
Utah	80	10
	81	294
Other		
Brazil	0	1
France	0	1
Hawaii	91	11
Turkey	0	1
	91	14

(1988b). Doggett *et al.* (1988) displayed observations of Moonwatches 1 and 2 on maps. However, observers who were clouded or rained out were recorded the same way as observers who had clear skies and yet did not see the Moon. Several observations from Moonwatch 4 were presented by di Cicco (1989).

In this paper tallies of observations from Moonwatches 2, 3, and 5 are presented in Tables I–III, grouped by state or country. The details of all observations for Moonwatch 4 are included in Table IV. The individual useful observations from Moonwatches 1 and 5 are also mapped in Figs. 4 and 8, respectively.

Some crescent observations not associated with any Moonwatch are also included in Table IV. The first three lines report minor corrections to a similar table that appeared in Schaefer (1988b). The remaining observations were either privately communicated to us or were taken from Cave (1911), Danjon (1932, 1936) or Bortle (1990). Not included in Table IV are certain published observations that Schaefer *et al.* (1992) have found to contain internal inconsistencies, to be contradicted by meteorological reports, or to be obviously misdated. We consider them to be provably mistaken. In particular, the report of Whitmell (1909) of an observation from 1885 must be

TABLE II
Moonwatch 3 (1989 April 6): Percentage of Observers Reporting Sightings

	Percentage	No. of observers
Atlantic		
Delaware	100	2
Florida	97	32
Georgia	100	2
Maryland	67	18
North Carolina	88	8
Pennsylvania	67	9
South Carolina	100	2
Virginia	50	12
Washington, DC	50	2
	79	87
Central United States		
Arkansas	100	1
Illinois	75	8
Indiana	100	2
Iowa	100	5
Louisiana	100	10
Michigan	83	12
Minnesota	100	17
Missouri	0	1
Mississippi	70	10
Ohio	67	3
Ontario	100	3
Wisconsin	100	2
	88	74
Caribbean		
Aruba	0	1
Haiti	0	1
	0	2
Western United States		
Alaska	100	3
Arizona	100	16
California	99	92
Colorado	92	13
Idaho	100	1
Kansas	67	3
Manitoba	100	5
North Dakota	100	1
Nebraska	100	1
Nevada	100	3
New Mexico	100	19
Oklahoma	100	7
Oregon	100	6
Texas	98	41
Utah	100	7
Washington	100	2
Wyoming	100	2
	98	222
Mexico		
Tijuana	100	1
S. L. Potosi	100	1
	100	2

misdated. Observations reported by Horner (1911) and Whitmell (1916) are contradicted by meteorological reports of cloudiness and rain; the observations may be misdated or spurious. Claims by inexperienced observers of extremely early sightings during Moonwatch 4 (reported by Durrani 1989, 1990) contain several severe internal errors. The report by di Cicco (1989) of an early observation made by experienced observers during Moonwatch 4 was actually an observation from 1 month earlier (see observation 236 in Table IV).

Table IV is constructed in a format similar to the table in Schaefer (1988b), with observations numbered in direct continuation of that table. Abbreviated column heads include E/M (evening or morning observation), Long. (longitude east of Greenwich), Alt. (altitude of site in feet), RH (relative humidity for the site's seasonal and diurnal average from Pearce and Smith (1984)), and k_A (aerosol extinction coefficient in 0.01 mag/airmass, cf. Schaefer (1988b)). The dates are local dates. The extinction data are from Husar (1988) and Husar and Holloway (1984), which give changes in atmospheric clarity in the past two decades resulting from increased air pollution. For observations 219–225, the estimates of the extinction coefficient have been increased to account for the volcanic eruption from El Chichon. The parenthetical V or I in column 8 specifies whether the crescent was visible or invisible with either a telescope or binoculars.

Included in Table IV are various quantities used by many of the crescent prediction criteria to be discussed in Section 7. The arc of light (ARCL) is the angle between the centers of the Sun and Moon. The arc of vision (ARCV) is the altitude of the center of the Moon above an ideal horizon plus the altitude of the center of the Sun below an ideal horizon; it is roughly equal to the altitude of the Moon at sunset. The difference in azimuth (DAZ) is between the azimuths of the centers of the Sun and Moon; a positive value indicates that the Moon is more southerly than the Sun. ARCL, ARCV, and DAZ, all in degrees, are calculated for the time of best visibility with no corrections for refraction or parallax. The age of the Moon (Age) is the time in hours from astronomical conjunction to best visibility, with a negative value indicating

TABLE III
Moonwatch 5 (1990 August 21): Percentage of Observers Reporting Sightings

	Percentage	No. of observers
Northeast		
Maine	0	4
Massachusetts	0	2
Michigan	0	1
Minnesota	0	1
New Hampshire	0	2
New York	100	1
Ontario	33	3
Quebec	0	3
Vermont	33	3
	15	20
Caribbean		
Barbados	100	1
Florida	33	3
Turks-Caicos	0	1
Grand Cayman	100	1
	50	6
Southwest		
Arizona	80	15
California	30	23
Colorado	50	2
Kansas	0	1
Nevada	50	2
Oklahoma	0	1
Texas	59	17
	51	61
Far South		
Hawaii	100	3
Baja	50	2
Guanajuato	100	1
	83	6

an old crescent. The moonset lag time (Lag) is the time between sunset and moonset in minutes. Values of the visibility parameter (R) and its 1σ uncertainty (DR) were calculated from Schaefer's model (see Section 7).

4. ESTABLISHING THE LUNAR DATELINES

The LDL is defined as the geographic locus of points on which the probability of sighting the crescent is 50%. Traveling east from the LDL, the probability gradually decreases to zero, while traveling west, the probability increases to nearly unity for clear skies. From our mass of observations over a broad geographic area, we can measure the variation of this probability by plotting the fraction of positive sightings as a function of position. In doing so, we can determine the position of the LDL and estimate the width of the zone of uncertainty.

The process is best illustrated by examining the observations from Moonwatches 2 and 3, the most massive Moonwatches. In Fig. 1 we have plotted the percentage of successful sightings as a function of longitude for two ranges of latitude for Moonwatch 2. Both graphs show the probability of sighting rising from near zero in the east to near unity in the west. However, the graph for the southern latitudes has a much sharper rate of change (5% for every degree of longitude near the LDL) than the graph for the northern latitudes (2.5% per degree of longitude). This difference is caused by the significant width of the latitude bands and by the LDL having a substantial east/west component for the northern latitudes.

In Fig. 2 we have plotted the percentage of successful sightings as a function of latitude for a single range of longitude for Moonwatch 2. Figure 3, which is similar in construction to Fig. 1, is based on data from Moonwatch 3.

The typical width of the observational zone of uncertainty can be estimated from Figs. 1–3. (However, the top of Fig. 1 should not be used because the LDL runs nearly parallel along the latitude band. Thus the generally east/west visibility factor is complicated by an increase of visibility from north to south.) In the bottom of Fig. 1, the frequency rises from zero to 90% in roughly 20° of longitude. In the top graph of Fig. 3, the extent in longitude from the LDL (50%) to where essentially all observers spot the Moon can be seen to be somewhat over 30°.

The results of the observational analysis are as follows:

Moonwatch 1. The observed LDL for Moonwatch 1 is somewhat uncertain because of the relatively small number of useful observations (see Doggett *et al.* 1988 or Schaefer 1988b). Nevertheless, it is clear that the LDL crosses the eastern United States roughly as a north/south line near 85° west longitude. It is unlikely that the real LDL is west of 95° since all sightings there are positive. From longitudes 75°–95° west, the negative sightings outnumber the positive sightings by 11 to 8, so that the real LDL is unlikely to be east of 75° west longitude. Therefore, we conclude that Moonwatch 1 had an observed LDL in the United States along longitude 85° west, with an uncertainty of under 10°.

Moonwatch 2. The observed LDL for Moonwatch 2 can be accurately defined because of the large number of observations. Three points where the fraction of positive sightings passes through 50% can be identified from Figs. 1 and 2. At latitude 30° north, the LDL has a longitude of 92 ± 2° west; at latitude 40° north, the LDL has a longitude of 107 ± 10° west; and at longitude 112.5° west, the LDL has a latitude of 43 ± 2° north. The uncertainty of 2° implies that we have measured the position of the LDL to within 140 miles. These three points are close to the LDL curve given in Doggett and Schaefer (1989),

TABLE IV
Individual Observations, Including Moonwatch 4 (1989 May 5)

No.	Year	M	D	E/M	JD (conj.)	Observer	Vis.	Lat.	Long.	Alt.	RH	k_A	ARCL	ARCV	DAZ	Age	Lag	$R \pm DR$	Y/N	(Sig)
								°	°				°	°	°	h	m			
106	1931	8	13	M	2426567.352	Danjon	I(V)	48.6	7.7	400	60	21	10.6	10.4	2.1	−16.6	71	−0.7±0.5	Y	(1)
122	1973	7	1	E	2441863.986	Austin	I(V)	−44.0	170.5	3900	80	10	10.6	8.6	−6.2	18.0	55	−0.2±0.3	Y	(1)
184	1987	4	28	E	2446913.566	Byrd	I	33.0	− 87.4	200	55	21	11.8	11.8	0.2	23.4	60	−0.4±0.5	Y	(1)
202	1885	12	7	E	2409882.053	DeCroupet	V	50.6	5.7	700	70	11	13.4	10.8	8.1	27.1	80	0.4±0.4	Y	(1)
203	1908	5	1	E	2418062.148	Bac	V	44.1	3.1	600	60	18	14.7	10.7	10.1	27.8	64	0.0±0.4	Y	(0)
204	1911	1	31	E	2419066.906	Cave	V	51.0	− 0.9	200	70	13	16.4	9.7	13.2	31.7	69	0.2±0.4	Y	(0)
205	1920	4	19	E	2422433.405	Triou	V	43.5	7.0	50	60	19	12.0	11.0	4.8	21.1	64	−0.4±0.4	N	(−1)
206	1932	4	7	E	2426803.556	Andreko +	V	50.0	36.2	400	60	16	18.0	16.9	6.1	39.8	111	2.0±0.2	Y	(9)
207	1933	3	27	E	2427157.640	Danjon	V	48.6	7.7	400	60	16	18.9	18.3	4.8	39.5	113	2.2±0.2	Y	(9)
208	1933	5	25	E	2427216.422	Tshernov	V	55.6	33.9	700	60	14	15.4	11.9	9.8	45.0	125	0.7±0.3	Y	(2)
209	1933	8	20	M	2427305.742	Danjon	V	48.6	7.7	400	60	20	12.4	11.2	5.5	−25.8	73	−0.1±0.4	N	(−0)
210	1933	9	18	M	2427335.265	Danjon	V	48.6	7.7	400	60	16	19.2	17.0	9.0	−37.9	103	2.1±0.2	Y	(9)
211	1934	3	16	E	2427512.007	Tshernov	V	55.6	33.9	700	60	12	15.8	15.1	4.7	28.6	107	1.6±0.2	Y	(7)
212	1934	5	14	E	2427571.021	Andreko +	V	50.0	36.2	400	60	17	14.6	13.1	6.5	29.6	102	0.8±0.4	Y	(2)
213	1934	5	14	E	2427571.021	Loreta	V	48.2	5.1	800	70	19	15.5	14.1	6.6	31.6	103	1.0±0.4	Y	(3)
214	1934	5	14	E	2427571.021	Tshernov	V	55.6	33.9	700	60	14	14.9	12.4	8.3	30.3	120	0.8±0.3	Y	(2)
215	1934	6	13	E	2427600.591	Tshernov	V	55.6	33.9	700	60	16	18.8	10.0	15.9	41.0	103	0.5±0.3	Y	(1)
216	1935	2	4	E	2427837.187	Loreta	V	48.2	5.1	800	75	17	14.8	13.8	5.4	25.0	87	1.0±0.3	Y	(3)
217	1935	4	4	E	2427896.008	Loreta	V	48.2	5.1	800	70	17	18.2	17.8	3.7	30.9	111	2.0±0.2	Y	(8)
218	1982	4	24	E	2445083.353	Stamm	V	37.2	− 84.1	1000	55	19	16.7	13.6	9.8	28.5	71	1.0±0.4	Y	(3)
219	1983	7	9	M	2445526.013	Stamm	V	37.2	− 84.1	1000	60	29	15.4	12.6	8.9	−26.4	72	0.1±0.5	Y	(0)
220	1983	11	5	E	2445643.432	Stamm	I(V)	37.2	− 84.1	1000	65	18	13.1	8.9	9.6	24.6	47	−0.8±0.5	Y	(1)
221	1983	12	5	E	2445673.019	Stamm	I(V)	37.2	− 84.1	1000	70	17	16.9	11.0	12.9	34.5	63	0.5±0.5	N	(−1)
222	1984	1	3	E	2445702.709	Stamm	I(I)	37.2	− 84.1	1000	70	14	8.8	5.0	7.3	17.5	29	−2.5±0.4	Y	(6)
223	1984	3	3	E	2445762.272	Stamm	I(V)	37.2	− 84.1	1000	60	17	14.2	11.7	7.9	29.6	59	0.4±0.4	N	(−1)
224	1984	5	1	E	2445821.657	Stamm	I(I)	37.2	− 84.1	1000	60	19	10.3	8.9	5.2	21.0	47	−1.2±0.5	Y	(3)
225	1984	11	23	E	2446027.457	Stamm	I(V)	34.0	− 81.0	200	65	17	13.3	7.8	10.7	23.6	43	−1.1±0.5	Y	(2)
226	1985	4	20	E	2446175.724	Stamm	I(I)	37.2	− 84.1	1000	55	19	8.7	8.0	3.5	19.1	41	−1.8±0.5	Y	(4)
227	1987	4	27	M	2446913.567	Stamm	I(I)	37.2	− 84.1	1000	70	25	7.8	5.5	5.6	−14.7	28	−3.3±0.7	Y	(5)
228	1987	6	26	E	2446972.734	Stamm	I(V)	37.2	− 84.1	1000	60	23	10.3	9.8	3.1	19.8	58	−1.1±0.5	Y	(2)
229	1987	9	23	E	2447061.631	Stamm	I(I)	37.2	− 84.1	1000	60	13	10.2	4.4	9.2	20.4	22	−2.5±0.4	Y	(6)
230	1988	1	19	E	2447179.727	Stamm	I(V)	32.2	−111.0	2560	40	5	12.1	9.7	7.3	19.8	49	0.4±0.3	N	(−1)
231	1988	4	16	E	2447268.001	Stamm	I(I)	37.2	− 84.1	1000	55	18	7.9	7.8	−1.2	12.5	41	−2.0±0.5	Y	(4)
232	1988	5	15	M	2447297.424	Stamm	I(I)	37.2	− 84.1	1000	60	22	7.7	7.2	2.8	−11.7	40	−2.5±0.5	Y	(5)
233	1988	6	13	M	2447326.885	Stamm	I(V)	37.2	− 84.1	1000	60	29	12.9	11.7	5.5	−23.4	69	−0.5±0.6	Y	(1)
234	1988	6	14	E	2447326.885	Stamm	I(I)	37.2	− 84.1	1000	60	29	9.2	9.1	1.5	16.0	54	−1.9±0.6	Y	(3)
235	1989	4	4	M	2447622.649	Hannigan	V	41.9	− 88.7	830	90	54	23.7	11.3	20.9	−40.2	61	−0.6±0.8	N	(−1)
236	1989	4	6	E	2447622.649	Shore +	V	34.0	−107.2	9900	35	40	14.1	14.1	−1.0	22.7	70	1.9±0.1	Y	( )

where the black dots on the map represent both observers who were clouded out and observers with clear skies who did not see the Moon.

Moonwatch 3. The observed LDL for Moonwatch 3 cannot be defined to the same precision as in Moonwatch 2 because we have no observations from sites (in the Atlantic) where the majority of observers would not see the Moon. Nevertheless, an analysis of Fig. 3 can yield positions for the LDL with reasonable accuracy. All prediction algorithms show the LDL to be closely north/south for the relevant latitudes. Therefore the frequency of sightings as a function of longitude should be the same in Fig. 3 as it is in the lower graph of Fig. 1. The difference between the two cases being merely an offset in longitude. We find that at latitude 40° north, the LDL has a longitude of 73 ± 3° west, while at latitude 30° north, it is at longitude of 66 ± 10° west. The larger error estimate for the southern point reflects the poor statistics for negative observations in Florida.

Moonwatch 4. The observed LDL for Moonwatch 4 is difficult to define accurately because there were few useful observations. The Moon was invisible to the naked eye at all sites, yet was barely visible with binoculars through various heroic efforts. Therefore, the observers were located east of the LDL, roughly where the probability for naked eye detection falls to near zero. We estimate that the LDL should lie roughly 40° west of the centroid of the observing sites. Since this centroid is near latitude 40° north and longitude 92° west, we place the LDL for this latitude at longitude 132° west, with an uncertainty of perhaps 30°. According to the prediction algorithms, the LDL should be roughly north/south in orientation for the latitude range of the continental United States. Therefore, the LDL at latitude 30° north will also be at longitude 132 ± 30° west.

Moonwatch 5. The observed LDL for Moonwatch 5 can be estimated for three regions with concentrations of reports. (1) Of the six observations from around the

TABLE IV—*Continued*

No.	Year	M	D	E/M	JD (conj.)	Observer	Vis.	Lat.	Long.	Alt.	RH	k_A	ARCL	ARCV	DAZ	Age	Lag	$R \pm DR$	Y/N	(Sig)
								°	°				°	°	°	h	m			
237	1989	5	5	E	2447651.992	Hunefeld +	I(V)	43.0	− 85.7	800	60	22	9.4	9.3	−0.3	13.4	57	−1.5±0.5	Y	(3)
238	1989	5	5	E	2447651.992	Pearson +	I(V)	39.7	−105.5	11000	50	4	10.0	9.9	−0.3	14.9	57	0.5±0.2	N	(−2)
239	1989	5	5	E	2447651.992	Victor	I(V)	42.7	− 84.8	850	60	20	9.3	9.3	−0.4	13.4	57	−1.4±0.5	Y	(3)
240	1989	5	5	E	2447651.992	Heaslip +	I(I)	42.7	− 84.8	850	60	20	9.3	9.3	−0.4	13.4	57	−1.4±0.5	Y	(3)
241	1989	5	5	E	2447651.992	Pearce +	I(V)	30.3	− 97.0	600	65	22	9.5	9.2	−2.5	13.7	46	−1.5±0.5	Y	(3)
242	1989	6	3	E	2447681.329	Krisciunas	I(I)	19.8	−155.5	13960	40	3	7.1	6.4	−3.0	9.4	30	−0.9±0.3	Y	(4)
243	1989	6	4	E	2447681.329	Arnold	V	50.8	− 1.0	0	75	27	14.3	11.4	8.6	25.0	97	−0.3±0.5	N	(−1)
244	1989	7	4	E	2447710.708	Harlan	V	37.4	−121.6	4210	70	11	24.1	16.2	18.0	47.3	92	2.7±0.2	Y	( )
245	1989	10	2	E	2447799.408	Schaefer +	V	36.1	−108.8	6500	30	5	34.7	13.6	32.0	75.8	71	3.2±0.1	Y	( )
246	1990	4	24	M	2448006.686	Bortle	I(V)	41.6	− 73.7	100	60	16	11.9	9.0	7.8	−18.7	50	−0.9±0.4	Y	(2)
247	1990	4	25	E	2448006.686	Bortle	I(V)	41.6	− 73.7	100	60	24	12.8	12.8	0.8	19.9	75	−0.0±0.5	Y	(0)
248	1990	4	25	E	2448006.686	Jones	V	37.7	−121.5	200	60	26	14.5	14.5	0.6	23.1	79	0.6±0.5	Y	(1)
249	1990	5	23	M	2448035.992	Bieda	V	31.6	−110.5	4500	10	5	15.0	13.0	7.5	−24.0	67	1.6±0.2	Y	(9)
250	1990	5	24	E	2448035.992	Bieda	I(V)	31.6	−110.5	4500	10	5	9.9	9.9	0.2	15.1	52	0.1±0.3	N	(−0)
251	1990	5	24	E	2448035.992	Bortle +	I(V)	34.2	−118.1	1740	40	12	10.2	10.2	0.8	15.5	56	−0.6±0.4	Y	(1)
252	1990	5	24	E	2448035.992	O'Meara	V(V)	34.2	−118.1	1740	40	12	10.2	10.2	0.8	15.5	56	−0.6±0.4	N	(−1)

Note. Year, M, and D give the local date of the observation. JD is the Julian date of conjunction. E/M specifies an evening or morning observation. Vis. indicates whether the Moon was sighted: V means it was visible; I, invisible; V or I in parentheses indicates the result of observations with binoculars or telescopes. Long. is the longitude measured positive east of Greenwich. Alt. is the altitude of the observer in feet above sea level. RH is the relative humidity, based on the site's seasonal and diurnal average (Pearce and Smith 1984). k_A is the aerosol extinction coefficient in 0.01 mag/airmass (cf. Schaefer 1988b), taken from Husar (1988) and Husar and Holloway (1984), which give changes in atmospheric clarity resulting from increased air pollution. ARCL is the arc of light, the angle between the centers of the Sun and Moon. ARCV is the altitude of the center of the Moon above an ideal horizon plus the altitude of the center of the Sun below an ideal horizon (roughly equal to the altitude of the Moon at sunset). DAZ is the difference between the azimuths of the centers of the Sun and Moon; a positive value indicates that the Moon is south of the Sun. (ARCL, ARCV and DAZ were calculated for the time of best visibility with no corrections for refraction or parallax.) Age is the interval from astronomical conjunction to best visibility, with a negative value indicating an old crescent prior to New Moon. Lag is the interval between sunset and moonset on the day of the observation. R, Schaefer's visibility parameter, and DR, its 1σ uncertainty, were calculated from Schaefer's model (see Section 7).

Caribbean (with average latitude 22° north and average longitude 76° west), half were positive sightings. Therefore the LDL is roughly at this average location, with an uncertainty (based on the half-width of the area of observations) of perhaps 6° in latitude. (2) Of the observations from longitudes 95°–100° west, the fraction of positive sightings is 100, 67, 25, and 0% for the latitude ranges 20°–25°, 25°–30°, 30°–35°, and 35°–40°, respectively. From these values, the LDL has a latitude of 32° north at longitude 98° west, with an uncertainty of roughly 2° in latitude. (3) Of the observations from longitudes 110°–124° west, the fraction of positive sightings is 53 and 18% for the latitude ranges 30°–35° and 35°–41°, respectively. From these values, the LDL has a latitude of 32.5° north at longitude 116° west, with an uncertainty of roughly 2° in latitude. The uncertainties in longitude can be deduced from the uncertainties in latitude, scaled by the measured slope of the LDL. Therefore, at latitude 30° north, the LDL has a longitude of 98 ± 8° west, while at latitude 32.5° north, the LDL has a longitude of 116 ± 14° west.

The positions of the observed LDLs for each of the five Moonwatches are tabulated in Table VIII and plotted in Figs. 4 through 8, where they are given for comparison with various prediction models (Sections 7 and 8).

5. FACTORS AFFECTING OBSERVATIONS

Our observational database enabled us to study a number of factors that are critical to the visibility of the lunar crescent. These may be categorized as atmospheric, optical, and human. Since our observation set includes naked-eye, optically aided visual, and photographic (short and long focus) observations, we were also able to examine explanations of why the Moon subtends less than the 180° of arc expected of a perfectly smooth satellite observed under perfect seeing conditions.

Atmospheric factors. The atmospheric seeing or steadiness is not a factor in crescent visibility, as will be discussed at the end of this section. Instead, atmospheric transparency, even in a "clear" or "cloudless" sky, is a crucial factor.

For Moonwatch 2, only 41% of observers near Los Angeles sighted the Moon, even though their neighbors in California and Arizona had an 88% rate of sightings. Many of the Los Angeles observers complained of bad smog conditions, and this is undoubtedly the reason for the low detection rate. Husar (1988), Husar and Holloway (1984), and Flowers *et al.* (1969) have shown that the extinction coefficient in summer within several hundred miles of Los Angeles is as bad as hazy sites in the eastern United States. For Moonwatch 3, however, the skies over

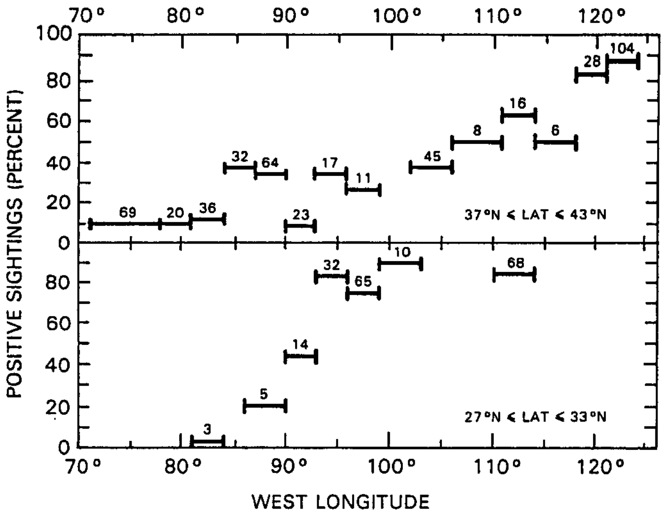

FIG. 1. Frequency of crescent sightings as a function of longitude, Moonwatch 2. As an observer moves west, the probability of spotting the Moon increases from zero to nearly unity. We have measured this relation by plotting the frequency of positive sightings as a function of longitude for two bands of latitude. The numbers next to each bar indicate the total number of reports from that region. Such a plot can be used to locate the longitude (and its uncertainty) of the LDL as the position where the frequency equals 50%. The plot is also useful for evaluating the width of the observational zone of uncertainty. In the top plot (for the latitude band 37°–43°), the observational zone of uncertainty is fairly wide because the LDL runs roughly east/west (cf. Fig. 5). In the bottom plot (for the latitude band 27°–33°), the frequency rises from zero to near unity over a small longitude range.

Los Angeles were unusually clear, because a brisk Santa Ana wind blew the smog away.

In general, we expect the presence of clouds to diminish the likelihood of spotting the crescent, primarily because the Moon will be hidden behind the clouds and never be seen. However, there are some subtle effects that might slightly change the probability of success if the Moon were seen through a hole in the clouds. One such effect is a reduction of glare, because the surrounding clouds are usually darker than the sky near the Moon. This is similar to the old myth that stars can be seen in daytime (cf. Hughes 1983). However, detailed calculations (Schaefer 1991b) show the reduction in surface brightness due to diminished glare would be 11%, even for a small hole and perfectly dark clouds. At the same time, physiological effects will offset the trivial gain in visibility from glare reduction (Martin 1923, Emerson and Martin 1925). As a test of these strong theoretical expectations, we have examined data for several regions with partly cloudy skies during Moonwatch 3. Combining observations from Ohio, Illinois, Oklahoma, and Colorado, the fraction of observers seeing the Moon with the naked eye was 63, 43, 35, and 0% when the reported sky conditions on the western horizon were clear, hazy, partly cloudy, and cloudy, respectively. Thus, from theoretical and observational points of view, the presence of clouds can only make the probability of a sighting lower.

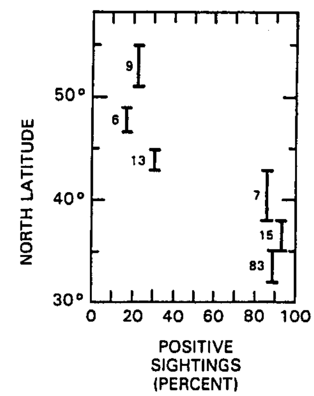

FIG. 2. Frequency of sightings as a function of latitude, Moonwatch 2. In the western United States, the LDL for Moonwatch 2 runs nearly east/west, so that a plot of frequency as a function of longitude (see top of Fig. 1) will not distinguish the position of the LDL with good accuracy. What is needed is a plot of frequency for some band roughly perpendicular to the LDL. This plot shows the frequency of crescent sightings as a function of latitude for the band of longitudes 110°–115° west. The LDL for this longitude band has a latitude of 43° with an uncertainty of 2°.

Optical factors. We would expect that an observer with binoculars would be more likely to spot the Moon than an observer with unaided vision. Since our data include instances when large numbers of observations from

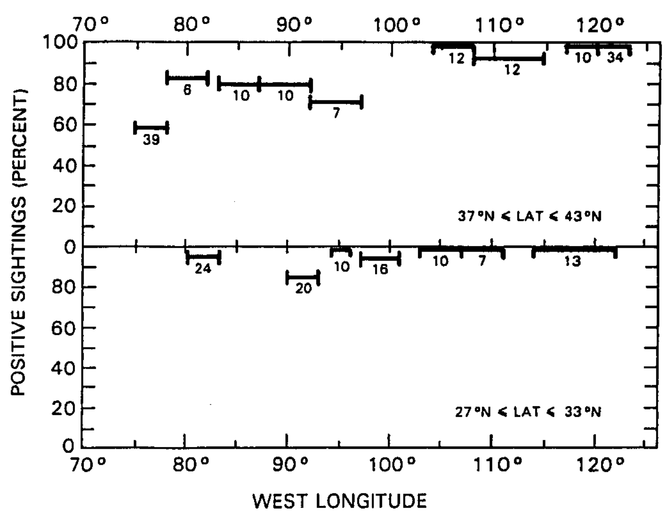

FIG. 3. Frequency as a function of longitude, Moonwatch 3. This plot is similar to that of Fig. 1. For the 37°–43° latitude band, the easternmost point has many observers and just over a 50% sighting rate. Hence the LDL is close to the coast at this latitude. For the 27°–33° latitude band, the 95% sighting fraction in Florida shows that the LDL is far off the coast.

TABLE V
Moonwatches 2 (14 July 1988) and 3 (6 April 1989): Percentages of Naked-Eye and Optically Aided Sightings

			Naked-eye		Optically aided	
Moonwatch	Latitude	Longitude	Percentage	No. of observers	Percentage	No. of observers
2	33–37	78–84	0	29	11	9
2	37–43	71–77	10	69	6	16
2	43–48	79–86	11	28	71	21
2	37–43	80–85	20	65	66	35
2	37–43	87–89	34	64	67	16
2	37–43	102–105	38	45	75	12
3	37–43	75–77	59	39	82	22
2	27–33	90–98	74	111	77	17
3	37–43	119–124	100	44	100	19

small regions were made with and without optical aid, we can study this effect quantitatively. Tallies for these regions are presented in Table V. The "Optically aided" tallies include only those reports explicitly stating that binoculars or telescopes were used. We find that while a naked-eye observer on the LDL has (by definition) a 50% chance of sighting the crescent, the probability of detection by an observer equipped with binoculars is roughly 75%. The probability of an observer with binoculars not sighting the Moon is approximately the square of the probability of a naked-eye observer not sighting the Moon.

Many observers commented that after vainly scanning the skies with the unaided eye, they located the Moon with binoculars, whereupon they immediately spotted the Moon with the unaided eye. Knowing where to look greatly increases the probability of detection, since day vision is 16 times more sensitive in the small area around the fovea (Minnaert 1954). This experience is similar to a search for Venus in the daytime sky. Twenty-five of the 162 observers who sighted the Moon with binoculars during Moonwatch 3, proceeded immediately to spot the Moon with the naked eye. Actually the number is likely to be much higher, since many observers did not separately quote times of visibility for aided and unaided vision.

Human factors. Successful sighting might depend to some extent on the observers' eyesight, experience, or age. Numerous reports from Moonwatch 2 provided information on these quantities. To control at least some variables, we present statistics in Table VI for five isolated regions of the country: southern Arizona (south of 35° latitude), northern California (36°–40° in latitude and west of 121° longitude), central Colorado (within 100 km of Denver), eastern Texas (east of 100° longitude), and southeastern Wisconsin (south of 45° latitude and east of 90° longitude). For each region, we give the fraction of positive sightings and the total number of useful observations.

Because relatively few people claimed to have good eyesight, the fraction of positive sightings for observers with good eyesight is fairly noisy. Combining all five areas to improve the statistics, we find that 82% of the observers with good eyesight spotted the Moon, while only 71% of the people with normal vision saw the crescent. The significance of this result is low, however, since only two or three sharp-eyed observers need to change their result for the percentages to match. Nevertheless, we feel that good eyesight provides a small but real improvement in the probability of detecting the crescent.

We expect that observers with better than average experience will have an improved chance of spotting the Moon. In the similar observational task of detecting faint stars through a telescope, Schaefer (1990a) found a small but significant improvement with experience. Since the observers of Moonwatch 2 covered the spectrum of observational experience, the statistics in Table VI are reasonably reliable. Four of the test regions show a significant increase in the probability of detection for more experienced observers. When all five regions are taken together, extra experience increases the detection fraction from 68 to 81%. This effect is also small but real.

Age is also correlated with detection success. Once again, because of small samples within age groups for each region, the noise is large and obscures any pattern. However, by combining all five regions and constructing larger age groups, it is possible to see significant trends. For the age group 20–59 years old, the detection rate is 73%, whereas for teenagers and children it is 54%, and for observers over 60 years old, only 53%. The lower rate of both young and old observers can be attributed to systematically low visual acuity for these age groups. The average visual acuity for a large population is nearly constant from the age of 14 to 62 years, with more than 90% of the population having 5/5 vision (based on visibil-

Table VI
Moonwatch 2 (14 July 1988): Percentage of Sightings According to Eyesight, Experience, and Age Groups

	Arizona		California		Colorado		Texas		Wisconsin	
	Percentage	No. of observers	Percentage	No. of observers	Percentage	No. of observers	Percentage	No. of observers	Percentage	No. of observers
Eyesight										
Normal	87	84	88	96	39	44	80	94	27	55
Good	100	8	75	4	—	—	86	7	33	3
Experience										
Normal	86	63	84	74	35	34	82	77	25	51
Good	93	29	96	26	50	10	75	18	43	7
Age										
4– 9	100	3	33	3	—	—	100	1	0	1
10–19	67	3	100	1	50	4	100	1	29	7
20–29	100	4	88	8	67	3	88	8	38	8
30–39	96	24	88	25	45	11	86	29	18	11
40–49	88	16	88	17	36	11	75	28	33	12
50–59	90	10	88	16	50	6	75	12	29	7
60–69	75	12	100	2	20	5	71	7	0	4
70–79	78	9	—	—	0	2	0	1	0	2
80–89	0	1	—	—	—	—	—	—	—	—

ity tested at a distance of 5 feet) or better. For both significantly younger and significantly older ages the population average eyesight is poorer, so that the majority of 7-year-olds and 69-year-olds have eyesight worse than 5/5 (Slataper 1950). An additional effect for young observers is that they tend to have less experience than their elders. We conclude that the age effect is real, significant, and probably caused by correlations of visual acuity and experience with age.

In any large observing campaign, some observers will make errors and fail to spot the Moon, even though it was trivially visible from their locality. These errors could be caused by bad vision, observing in the wrong direction or at the wrong time, or having a high horizon. From Moonwatch 2, we discovered the importance of such errors, because we have five reports that the Moon was not even spotted on the day *after* the Moonwatch, despite clear skies. We divide these errors into two classes. A positive error occurs when an observer mistakenly claims to see the crescent; a negative error is when an observer mistakenly does not see the Moon.

A limit on the fractional rate of negative errors can be estimated from our data for Moonwatch 2. Of 520 negative reports, 5 were made by observers who missed an easy sighting on the following night. We suspect the rate of negative errors is greater (and probably much greater) than 1%.

The fractional rate of negative errors can be estimated from Moonwatch 3. All modern prediction algorithms place the western edge of the zone of uncertainty far east of 95° west longitude. Indeed, many western observers complained that visibility was trivial. Any observer in a western state who did not see the Moon despite clear skies can be considered to have made a negative error. Of the 213 observers in western states, only 4 failed to spot the Moon. Since this Moonwatch was composed of observers recruited through *Sky & Telescope*, they are likely to have more experience than the general population. Yet, they still had a 2% error rate.

The fractional rate of positive errors can be estimated from Moonwatch 5. All modern prediction algorithms place northeastern North America well outside the northern edge of the zone of uncertainty. Any observer in the northeast who claimed to have sighted the Moon must have been mistaken. Of the 20 observers in the northeast, 3 reported sighting the Moon. In all three cases, the reported time of sighting, orientation of the horns, and direction of the Moon were grossly in error. The large errors in reported details confirm that these three observations were positive errors. For our small sample from Moonwatch 5, the positive error rate is 15%.

McNally effect. During Moonwatch 3, the crescent did not appear to extend from the north pole to the south pole. That is, the lighted arc did not appear to subtend an angle of 180° from the disk center as to be expected if the Moon were perfectly smooth and seeing were negligible (McNally 1983). Thirty-five observers reported estimates of the arc length, while 35 photographs were of quality sufficient to allow measurements. These data can be subdivided into estimates with the unaided eye, visual estimates using binoculars, measurements of camera photographs (focal lengths under 300 mm), and measurements of telescopic photographs (focal lengths over 500 mm). For these classes the average subtended angles were 116°, 119°, 109°, and 113°, respectively, with rms deviations of nearly 15°. If McNally's seeing effect is dominant, then the arc length should vary with the angular scale for image smearing. However, image smearing will result from *both* atmospheric and instrumental effects. For direct vision, a convolution of the atmospheric smearing function (say 5″ in diameter near the horizon) and the eye's resolution (42″ in diameter on average, Blackwell 1946) will result in a total smearing with a characteristic size of nearly 42″. The binocular and photographic observations have resolutions much smaller than the seeing disk size, for a total smearing of perhaps 5″. Thus McNally would predict that measurements with the unaided eye should be significantly shorter than the other measurements. Since this is not seen in the data, we conclude that other phenomena must dominate over McNally's effect (Schaefer 1991a). In particular, Schaefer (1991a) demonstrates that all available data are consistent with the ends of the horns being invisible merely because they are below the visual threshold (i.e., the signal-to-noise ratio is too small for detection).

6. OBSERVATIONAL LUNAR CALENDARS

The atmospheric and personal factors discussed above provide some insight into the calendrical problems faced by societies that have used observational lunar calendars. Two points that are particularly relevent.

First, a significant fraction of observers will claim to have sighted the crescent even when it is impossible (e.g., when the Moon is below the horizon). Honest observers may make honest mistakes, for there are many objects in the sky (e.g., wisps of cloud or aircraft illuminated by the Sun) that can be mistaken for a crescent. Furthermore, the power of an observer's imagination (particularly that of an inexperienced observer) is undoubtedly a significant factor in false sightings. Our studies show that the rate of false sightings is 15%, i.e., if 100 observers try to determine the start of a lunar month by direct observation, 15 honest people, on average, will claim to have seen the crescent. Since observations often begin one or more evenings before actual first visibility (and perhaps even before conjunction, when the Moon is by definition an old Moon), it is likely that the month will mistakenly start early on the basis of honest mistakes. We know from experience that this happens. Historical dates recorded on lunar calendars undoubtedly are affected by this bias.

Second, there are reliable grounds for rejecting claims of extremely early sightings. Visual observation of the Moon is impossible when the Moon is too close to the Sun. Although the threshold of visibility depends on specific circumstances, the Danjon limit (i.e., the Moon is invisible within 7° of the Sun) is valid for all cases (Schaefer 1991). However, there are additional constraints that will further restrict crescent visibility. In practice, the Moon has never been sighted within 13.4 hr of conjunction. Even this record observation was made by a highly skilled, highly experienced, and highly prepared observer who used high-power binoculars on a steady mount. Any claimed sighting of a Moon that is much younger than this record, especially by casual observers or without optical aid, should be treated with skepticism.

Reports of extremely early sightings by inexperienced observers are often accompanied by details concerning the position and orientation of the crescent as well as the direction to the Moon that are at odds with calculations. Such reports need not be the basis for maintaining a lunar calendar.

7. PREDICTION MODELS

Prediction models specify an inequality that must be satisfied for visibility to be predicted. We have selected 13 such models, which can be divided into four groups according to historical and methodological considerations: ancient, medieval Islamic, modern empirical, and modern theoretical. Other algorithms have been used (for example by the ancient Jews), but we are not aware of their details and so cannot include them in this paper. The ancient criteria, which date back at least to the Babylonian civilization, are based on either the age of the Moon or the lag time from sunset to moonset. The medieval Islamic criteria originated between the 8th and 12th centuries A.D. in the Middle East. They are based on the difference between the ecliptic longitudes of the Moon and Sun. The modern empirical criteria are based on rules for the altitude and azimuth of the Moon at the time of sunset. The modern theoretical model is a theory derived from fundamental equations of astronomy, meteorology, and physiology.

The criteria of these algorithms are summarized in Table VII. In this table, $\Delta\lambda$ is the difference in ecliptic longitude between the Sun and Moon, and β is the ecliptic latitude of the Moon. These quantities were originally specified as functions of the time of year. In Table VII, however, we give only the values that are relevant to the five Moonwatches of this study.

The modern theoretical model of Schaefer requires additional comment. It is based on a calculation of the quantity $R = \max[\log(B_{moon}/B_{th})]$, where B_{moon} is the actual total brightness of the Moon and B_{th} is the total brightness required for visibility at various times throughout twilight under the given conditions. Associated with R is its standard deviation DR. The primary uncertainty in the evalua-

TABLE VII
Historical Prediction Criteria

Algorithm	Date	Ref.	Criterion
Ancient			
Unknown	Ancient	1	AGE > 24 hr
Babylonians	>4th century B.C.	1	LAG > 48 min
Medieval Islamic			
Ya'qub ibn Tariq	8th century A.D.	2	LAG > 48^m & ARCL > 11.25° LAG > 40^m & ARCL > 15°
Muhammad ibn Musa al-Khwarizmi	9th century	3, 4	$\Delta\lambda + \beta >$ 6.4°, 10.5°, 5.6°, 4.3°, 22.6°
Abu Jafar al-Khazin	10th century	4	$\Delta\lambda >$ 9.2°, 12.8°, 8.8°, 8.4°, 26.7°
Muhammad ibn Ayyub al-Tabari	11th century	4	$\Delta\lambda >$ 9.0°, 11.8°, 8.8°, 8.2°, 25.3°
Al-Fahhad	12th century	5	$\Delta\lambda >$ 9.0°, 8.5°, 8.0°, 7.0°, 8.9°
Moses ibn Maimon (Maimonides)	12th century	2	$\Delta\lambda >$ 9.9°, 9.5°, 10.1°, 9.3°, 14.9°
Modern empirical			
J. K. Fotheringham	1910	6	ARCV > f(DAZ), f(0°) = 12°, f(20°) = 10°
E. W. Maunder	1911	7	ARCV > f(DAZ), f(0°) = 11°, f(20°) = 6°
M. Ilyas	1984	1	ARCV > f(DAZ), f(0°) = 10°, f(20°) = 7°
B. D. Yallop	1980s	8	ARCV > f(w), w = crescent width DAZ = 0°, f(0.31') = 11.7°
Modern theoretical			
B. E. Schaefer	1988	9	$R > 0$, $R = \max[\log(B_{moon}/B_{th})]$

Note. References: 1, Ilyas (1984); 2, King (1988); 3, Kennedy and Janjanian (1988); 4, Hogendijk (1988); 5, King (1987); 6, Fotheringham (1910); 7, Maunder (1911); 8, Yallop (1987–1989); 9, Schaefer (1988a,b, 1990a,b).

tion of R is the aerosol extinction coefficient, which is calculated from average correlations with altitude, latitude, longitude, relative humidity and time of year. Additional aerosol extinction information is available in the form of monthly averages for roughly 250 sites over most of the world. Many of the primary equations for the modern theoretical model are presented in Schaefer (1993).

8. TESTING THE MODELS

In Table VIII we have tabulated data for comparing the observed and predicted LDLs for each of the five Moonwatches. In each case, west longitudes of an LDL are given for two reference latitudes. For the first 10 models, *The Astronomical Almanac* and associated computerized almanacs (Kaplan *et al.* 1986, Carroll 1988, Bangert *et al.* 1992) were used in calculating the LDLs. Relevant parameters were calculated for the time of apparent sunset. The time of apparent moonset was used for determining the moonset lag, though lunar positions for evaluating other quantities were calculated without refraction or parallax corrections. Longitudes of the LDLs for the first 10 criteria are presented to the nearest 5°.

LDLs for the recent models are taken from Ilyas (1984), Yallop (1987–1990), and Schaefer (1990b). In addition to being tabulated in Table VIII, the LDLs for these models are plotted in Figs. 4 through 8. Yallop (1987–1990) only gives the threshold for DAZ = 0°. We have used this value as the easternmost point of a parabola-like curve defining the LDL (see for example Ilyas 1984). For Schaefer's model, the predicted LDL is the locus of points for which the value of R is zero.

In Table IX we tabulate the bias, average deviation, and extreme deviation for each model as determined from data in Table VIII. We also give the total width of the zone of uncertainty as deduced from the 201 observations tabulated in Schaefer (1988b). The four quantities in Table IX can be used to judge the reliability of the various models.

The two ancient criteria are both of very poor accuracy. Since their zones of uncertainty cover the entire world, confident predictions can never be given.

The medieval Islamic criteria are not much better in that most of them also can never make a confident prediction for any location in the world. Ibn Tariq's algorithm fares best. Even so, its zone of uncertainty covers a whole hemisphere.

The modern empirical criteria are definitely better than the medieval criteria, although they still have a zone of uncertainty that spans over 100° in longitude. The best of them is that of Yallop.

Schaefer's modern theoretical model proves to be the best of all the algorithms tested. Bias, mean error, maximum error, and width of the zone of uncertainty are about a factor of two smaller than those of other models. Since only Schaefer's model accounts for local extinction conditions, this result is not surprising.

TABLE VIII
Observed and Predicted Lunar Date Lines

	West longitude of lunar date line									
Moonwatch:	1 (28/4/87)		2 (14/7/89)		3 (6/4/89)		4 (5/5/89)		5 (21/8/90)	
Latitudes:	30°N	40°N	30°N	40°N	30°N	40°N	30°N	40°N	30°N	32.5°N
Observed	85 ± 10	85 ± 10	92 ± 2	107 ± 10	66 ± 10	73 ± 3	132 ± 30	132 ± 30	98 ± 8	116 ± 14
Age	105	100	40	35	140	135	260	260	−90	−90
Babylonian	45	5	30	10	35	−5	130	130	220	260
Ibn Tariq	75	70	25	20	45	40	160	155	130	150
Al-Khwarizmi	−55	−60	20	10	−80	−85	10	10	215	210
Al-Khazin	25	20	95	85	0	0	115	115	335	330
Al-Tabari	20	15	60	55	0	0	110	110	290	290
Al-Fahhad	20	15	−45	−55	−20	−20	80	80	−15	−10
Maimonides	50	20	15	20	35	10	140	140	−15	−5
Fotheringham	100	100	100	160	70	60	180	185	130	185
Maunder	70	65	25	45	45	35	150	155	5	65
Ilyas	45	40	15	50	35	25	145	135	40	64
Yallop	88	89	88	147	61	55	172	167	117	141
Schaefer	97	77	81[a]	84[a]	72	67	155[b]	145[b]	102	112

Note. Data in degrees.

[a] The tabulated predictions are based on extinction coefficient data from the 1960s (Flowers *et al.* 1970). However, Husar (1988) has demonstrated a substantial increase in the summer extinction coefficients over the eastern United States from the 1960s to the 1980s. If the modern extinction data is used, the tabulated values will change to 97° and 98° west longitude for 30° and 40° north latitude, respectively.

[b] In many cases, the regions of crescent visibility and invisibility may be separated by a simple curve (the LDL). However, in some cases, for example where a region of good atmospheric clarity lies somewhat to the east of the "average" LDL, the dividing line may not be simply connected. The tabulated values from the Schaefer model are for the sea level average LDL, even though many sites in the mountainous western states are predicted to easily spot the Moon.

Schaefer's model is unique in that it has a statistical estimate of the accuracy of any visibility claim. That is, the value of *R*/*DR* represents a measure of how many standard deviations the Moon is away from the threshold of visibility. For example, if the prediction is that the Moon should be visible with an *R*/*DR* value of 1.0, there will be a 16% probability that the prediction will be wrong. Our data enable us to test the statistical nature of *R*/*DR*. For the 252 observations in Schaefer (1988b) and Table IV of this paper, the observations with N in the next to last column should fall off as a Gaussian with zero center

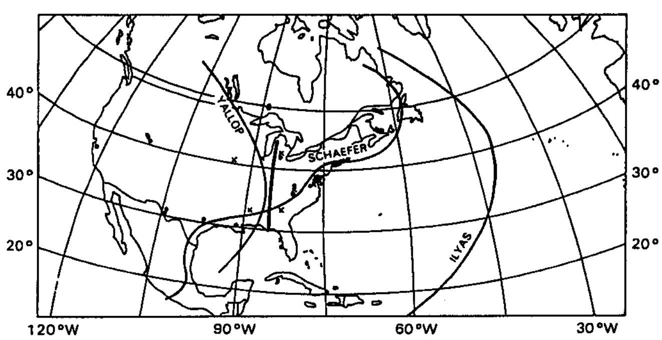

FIG. 4. Lunar date line, Moonwatch 1. The filled circles indicate observers for whom the Moon was visible to the naked eye; crosses indicate observers for whom the Moon was not visible. The observed LDL is roughly at longitude 85 ± 10° west, as indicated by the thick unmarked line. The predicted LDLs for the models of Ilyas, Yallop, and Schaefer are indicated by the labeled curves. The models of Yallop and Schaefer are consistent with the observations, whereas the model of Ilyas predicts an LDL a substantial distance to the east.

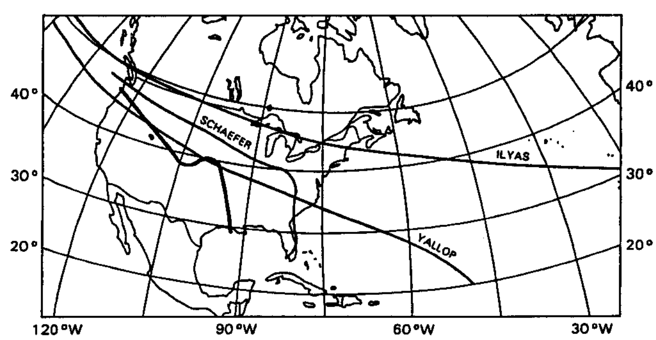

FIG. 5. Lunar date line, Moonwatch 2. The position of the observed LDL (the thick unlabeled curve on the map) was measured to an accuracy of roughly 140 miles because of the large number of reports. The predicted LDLs of the Yallop and Schaefer models both agree with observations, while the predicted LDL of Ilyas is substantially to the east.

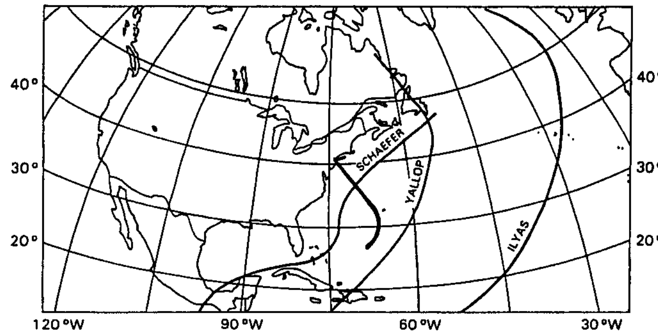

FIG. 6. Lunar date line, Moonwatch 3. The position of the observed LDL (the thick unlabeled curve on the map) was deduced from the fraction of observers on the east coast of America that did not see the Moon. The predicted LDLs of the Yallop and Schaefer models both agree with observations, while the Ilyas' predicted LDL is substantially to the east.

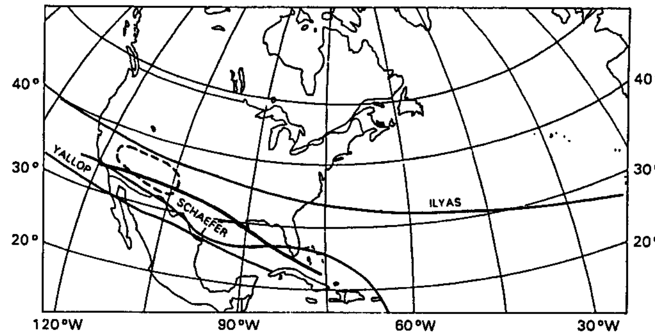

FIG. 8. Lunar date line, Moonwatch 5. The position of the observed LDL (the thick unlabeled curve on the map) was deduced from the rate of positive sightings in the Caribbean and the southwest. As in Fig. 7, the predicted LDL from Schaefer's model is not simply connected, in that high-altitude sites in the southwestern United States were predicted to see the crescent. The predicted LDLs of the Yallop and Schaefer models both agree with observations, while the Ilyas' predicted LDL is substantially to the east.

and unity standard deviation. In fact, 22% have *R*/*DR* less than 0.5 (while 38% is predicted), 78% have *R*/*DR* less than 1.5 (while 86% is predicted), and 97% have *R*/*DR* less than 2.5 (while 99% is predicted). These results suggest that *R*/*DR* is a slightly conservative estimate of the true reliability of the predictions.

9. CONCLUSIONS

We have collected 1534 useful observations of lunar crescent visibility from five Moonwatches held in North America during the period 1987–1990. Combining these with observations collected from the literature or communicated to us, we have examined a database that is an order of magnitude larger than has ever been previously utilized. From these observations we have determined the Lunar Date Line for each of the Moonwatches. For Moonwatches 2 and 5, the LDL was determined to an accuracy of 140 miles. Around the LDL is an observational zone of uncertainty having a half width of roughly

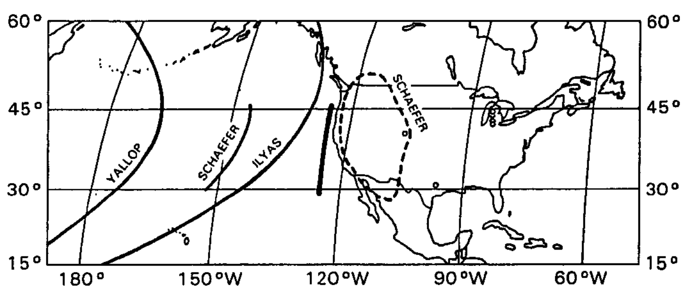

FIG. 7. Lunar date line, Moonwatch 4. Only six observers were not clouded out; however, five of six spotted the very young Moon. The one sighting with unaided eyes (the filled circle) was made at an altitude of 10,600 feet, in the clear skies characteristic of the southwestern United States. The Moon was sighted with binoculars from four other sites (unfilled circles). At one site in Michigan (the cross), the Moon was not seen even through binoculars. The position of the observed LDL is estimated to be at longitude 125° west (indicated by the thick unlabeled line on the map), but the uncertainty is roughly 15°. The predicted LDLs of Ilyas and Schaefer are both consistent with the data, whereas the predicted LDL of Yallop is substantially to the west. The model of Schaefer does not necessarily predict that regions of visibility will be separated from the regions of invisibility by a simple curve. In the case of Moonwatch 4, the "average LDL" is predicted to lie in the Pacific Ocean. Nevertheless, high altitude sites in the southwestern United States (indicated by a dashed curve) should spot the Moon.

TABLE IX
Deviations of Models from Observations

Model	Bias	Mean error	Maximum error	Width of LDL[a]
Ancient				
Age	−9	93	206	>360
Babylonian	−13	66	144	>360
Medieval Islamic				
Ibn Tariq	−9	37	87	—
Al-Khwarizmi	−88	121	156	—
Al-Khazin	13	77	236	—
Al-Tabari	−4	78	194	—
Al-Fahhad	−96	96	162	—
Maimonides	−58	61	121	—
Modern empirical				
Fotheringham	30	30	71	54
Maunder	−33	42	95	54
Ilyas	−39	42	77	54
Yallop	14	19	40	54
Modern theoretical				
Schaefer	1	11	23	24

Note. Data in degrees.
[a] From the 201 observations tabulated in Schaefer (1988b).

30° in longitude. Within this zone the Moon may or may not be sighted depending on local conditions.

We find that the experience and eyesight of the observer have small but significant effects on the probability that the crescent will be spotted. The observer's age is also correlated with detection frequency. We believe that this results from the correlation of age with experience and eyesight. Binoculars offer a large advantage for sighting the Moon. Observers with binoculars located on the LDL will see the Moon roughly 75% of the time, compared with 50% for naked-eye observers.

Previously, the youngest Moon ever seen with unaided vision was 15.4 hr by J. Schmidt; with optical aid the record was 14.9 hr by R. Moran (Schaefer 1988b). The record for observations with optical aid was broken by several groups during Moonwatch 4, with the new record being 13.4 hr by R. Victor (Table IV, observation 239). We note that the naked-eye sighting by S. O'Meara (Table IV, observation 252), at an age of 15.5 hr, is very close to Schmidt's record.

We find that the length of the arc subtended by the crescent was the same, whether estimated by the naked eye or with binoculars, or measured from photographs made with short focal length optics (cameras) or long focal length optics (telescopes). This demonstrates that the shortening of the crescent cannot be due to effects of atmospheric turbulence and seeing.

For a group of experienced observers, the percentage who failed to sight the Moon when it should have easily been spotted is roughly 2%. The rate of positive errors, when an observer erroneously claims a sighting, is 15%. This result has important implications for observational lunar calendars that are based on many observers looking for the Moon. If 100 observers look for the crescent, roughly 15 will mistakenly (yet honestly) claim to see the Moon. Therefore, lunar months based on a few positive sightings from a large number of observers will invariably and mistakenly start early.

In tests of models for predicting lunar visibility, the ancient and medieval algorithms fared least well, with zones of uncertainty typically covering most of the world. The altitude/azimuth criteria by Fotheringham, Maunder, Ilyas, and Yallop proved to be reasonably accurate, with the particular implementation by Yallop being better than the other three. The model by Schaefer yielded significantly better predictions than any other algorithm.

ACKNOWLEDGMENTS

We thank the many observers who have contributed reports to us. May they enjoy many evenings of pleasant sky watching. Drs. Dean Ahmad and Ken Seidelmann contributed many useful ideas, comments, and assistance. Ms. Jennifer Weeks provided assistance with calculations. We also thank Ms. Gail Cleere, formerly of the U.S. Naval Observatory Public Affairs Office, and the many newspapers, magazines, and television stations that alerted observers to the Moonwatches.

REFERENCES

BANGERT, J. A., *et al.* 1992. *Multi-year Interactive Computer Almanac* USNO, Washington, DC.

BLACKWELL, H. R. 1946. Contrast thresholds of the human eye. *JOSA* **36,** 624–643.

BORTLE, J. E. 1990. April's old and young Moon. *Sky & Tel.* **80,** 215.

BRUIN, F. 1977. The first visibility of the lunar crescent. *Vistas Astron.* **21,** 331–358.

CARROLL, T. S. 1988. *Floppy Almanac.* USNO, Washington.

CAVE, C. J. P. 1911. Early visibility of the new Moon. *Observatory* **34,** 121.

DANJON, A. 1932. Jeunes et Vieilles Lunes. *L'Astronomie* **46,** 57–66.

DANJON, A. 1936. Le Croissant Lunaire. *L'Astronomie* **50,** 57–65.

DI CICCO, D. 1989. Breaking the new-Moon record. *Sky & Tel.* **78,** 322–323.

DOGGETT, L. E., AND P. K. SEIDELMANN 1988. Calculating and observing the crescent Moon. In *Proceedings of the Lunar Calendar Conference* (I. Ahmad, Ed.), pp. 10-1-12. I.I.I.T., Herndon.

DOGGETT, L. E., P. K. SEIDELMANN, AND B. E. SCHAEFER 1988. Moonwatch—July 14, 1988. *Sky & Tel.* **76,** 34–35.

DOGGETT, L. E., AND B. E. SCHAEFER 1989. Results of the July Moonwatch. *Sky & Tel.* **77,** 373–375.

DOGGETT, L. E., AND B. E. SCHAEFER 1990. Moonwatch—August 21st. *Sky & Tel.* **80,** 174.

DURRANI, M. N. 1989. New world record for youngest naked eye crescent Moon sighting. *J. R. Astron. Soc. Can.* **83,** 34–6.

DURRANI, M. N. 1990. A still younger Moon. *Sky & Tel.* **79,** 582.

EMERSON, S. A., AND L. C. MARTIN 1925. The photometric matching field. *Proc. R. Soc. Series A* **108,** 483–500.

FLOWERS, E. C., R. A. MCCORMICK, AND K. R. KURFIS 1969. Atmospheric turbidity over the United States, 1961–1966. *J. Appl. Meteorol.* **8,** 955–962.

FOTHERINGHAM, J. K. 1910. On the smallest visible phase of the Moon. *Mon. Not. R. Astron. Soc.* **70,** 527–531.

HOGENDIJK, J. P. 1988. Three Islamic lunar crescent visibility tables. *J. Hist. Astron.* **19,** 29–44.

HORNER, D. W. 1911. Early visibility of the new Moon. *JBAA* **21,** 162–163; **21,** 203; **22,** 344–345.

HUGHES, D. W. 1983. On seeing stars (especially up chimneys). *Q. J. R. Astron. Soc.* **24,** 246–247.

HUSAR, R. B. 1988. *Trends of Seasonal Haziness and Sulfur Emissions Over the Eastern United U.S.* Report to E.P.A., CR813357.

HUSAR, R. B., AND J. M. HOLLOWAY 1984. The properties of climate and atmospheric haze. In *Hygroscopic Aerosols* (L. H. Ruhnke and A. Deepak, Eds.), pp. 129–170. Deepak, Hampton.

ILYAS, M. 1978. Visibility of the new Moon: Astronomical probability. *J. Malaysian Br. R. Asiatic Soc.* **51**(II), 58–68.

ILYAS, M. 1981. Lower limit of w in the new Moon's fist visibility criterion of Bruin and its comparison with the Maunder criterion. *Q. J. R. Astron. Soc.* **22,** 154–160.

ILYAS, M. 1984. *Islamic Calendar, Times & Qibla.* Berita, Kuala Lumpur.

KAPLAN, G. H., T. S. CARROLL, L. E. DOGGETT, AND P. K. SEIDELMANN 1986. *Floppy Almanac.* USNO, Washington, DC.

KENNEDY, E. S., AND M. JANJANIAN 1965. The crescent visibility table in Al-Khwarizmi's Zij. *Centaurus* **11**, 73–78.

KING, D. A. 1987. Lunar crescent visibility predictions in medieval Islamic Ephemerides. Preprint.

KING, D. A. 1988. Ibn Yunus on lunar crescent visibility. *J. History Astron.* **19**, 155–168.

MARTIN, L. C. 1923. The photometric matching field. *Proc. R. Soc. Series A* **104**, 302–315.

MAUNDER, E. W. 1911. On the smallest visible phase of the Moon. *JBAA* **21**, 355–362.

MCNALLY, D. 1983. The length of the lunar crescent. *Q. J. R. Astron. Soc.* **24**, 417–429.

MINNAERT, M. 1954. *The Nature of Light and Colour in the Open Air.* Dover, New York.

PEARCE, E. A., AND C. G. SMITH 1984. *World Weather Guide* Times, New York.

SCHAEFER, B. E. 1988a. An algorithm for predicting the visibility of the lunar crescent. In *Proceedings of the Lunar Calendar Conference* (I. Ahmad, Ed.), pp. 11-1-12. I.I.I.T., Herndon.

SCHAEFER, B. E. 1988b. Visibility of the lunar crescent. *Q. J. R. Astron. Soc.* **29**, 511–523.

SCHAEFER, B. E. 1990a. Telescopic limiting magnitudes *PASP* **102**, 212–229.

SCHAEFER, B. E. 1990b. *LunarCal.* Western Research Company, Inc., 2127 E. Speedway, Suite 209, Tucson, AZ 85719.

SCHAEFER, B. E. 1991a. Length of the lunar crescent. *Q. J. R. Astron. Soc.* **32**, 265–277.

SCHAEFER, B. E. 1991b. Glare and celestial visibility. *PASP* **103**, 645–660.

SCHAEFER, B. E., I. AHMAD, AND L. E. DOGGETT 1992. Records for young Moon sightings. *Q. J. R. Astron. Soc.* **34**, 53–56.

SCHAEFER, B. E. 1993. Astronomy and the limits of vision. *Vistas Astron.*, in press.

SLATAPER, F. J. 1950. Age norms of refraction and vision. *Arch. Ophthalmol.* **43**, 466–481.

WHITMELL, C. T. 1909. Early visibility of the crescent Moon. *JBAA* **19**, 254–255.

WHITMELL, C. T. 1916. Early visibility of the Moon. *JBAA* **27**, 36–38.

YALLOP, B. D. 1987–1990. *Earliest Sighting of the New Moon in 1987, 1988, 1989, and 1990.* RGO Astronomical Information Sheet, Nos. 50, 52, 55, and 56.